Engineering Notes

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Simple Rate-Independent Model for Damage

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Nomenclature

t	= time
ϵ	= strain (change in length/original length)
ϵ_m	= maximum strain
ξ	= ϵ/ϵ_m relative strain
σ	= stress (force/original area)

Introduction

A MAJOR flaw in most linear and nonlinear theories of material behavior lies in the neglect of internal damage. In rocket propellants, for example, such damage is caused, among other things, by the rupturing of bonds between the rubber matrix and the embedded particles.

To overcome this deficiency Mullins[1] proposed a theory in which the past maximum stress plays a major role, Farris and Fitzgerald developed a theory in which damage manifests itself through certain L_p norms of the strain history, and Quinlan[3] introduced a damage parameter whose behavior is governed by a nonlinear differential equation. One of the chief problems with the theories is their complicated nature, a factor that renders intuitive insight difficult and thereby results in a complicated program of experimental verification.

Our purpose here is to present a simple theory that isolates the main qualitative features of materials undergoing internal damage. The measure of damage is the maximum value of strain encountered in the loading process, and the theory is based on a constitutive equation given the current stress as a function of current strain and damage.

The Model

For convenience, attention is confined to *uniaxial tension*; that is, to strains $\epsilon(t)$ that are *nonnegative* at all times and vanish at all times prior to $t=0$. The chief hypothesis is that the current damage is completely characterized by the *maximum strain* (cf. Farris[4])

$$\epsilon_m(t) = \max \epsilon(\tau), \quad 0 \leq \tau \leq t \tag{1}$$

We assume that the stress $\sigma(t)$ is given by a constitutive equation of the form

$$\sigma(t) = g(\epsilon(t), \epsilon_m(t)) \tag{2}$$

and hence depends only on the current values of strain and damage. Of course such an equation is *rate independent*.

If the maximum strain occurs at the present time, then

$$\epsilon_m(t) = \epsilon(t) \tag{3}$$

and Eq. (2) reduces to

$$\sigma = G(\epsilon_m) = g(\epsilon_m, \epsilon_m) \tag{4}$$

The stress-strain curve

$$\sigma = G(\epsilon_m) \tag{5}$$

is called the *virgin curve* and is traced out in an experiment with monotonically increasing strain.

Using the virgin curve Eq. (2) can be rewritten, for arbitrary loading, as

$$\sigma = F(\xi, \epsilon_m) G(\epsilon_m) \tag{6}$$

with $\xi = \epsilon/\epsilon_m$ the relative strain and

$$F(\xi, \epsilon_m) = \frac{g(\xi, \epsilon_m, \epsilon_m)}{G(\epsilon_m)} \tag{7}$$

The function $F(\xi, \epsilon_m)$ (of ξ) is called the damage curve at the damage level ϵ_m. Note that

$$F(1, \epsilon_m) = 1 \tag{8}$$

In many situations of interest $F(\xi, \epsilon_m)$ is independent of ϵ_m:

$$F(\xi, \epsilon_m) = F(\xi) \tag{9}$$

when this is so $F(\xi)$ is called the *master damage curve*, and Eq. (6) reduces to

$$\sigma = F(\xi) G(\epsilon_m) \tag{10}$$

Experimental Results

To demonstrate the usefulness of the concepts developed, a simple test was performed with a highly filled solid propellant (TP-H1011-86% solids).

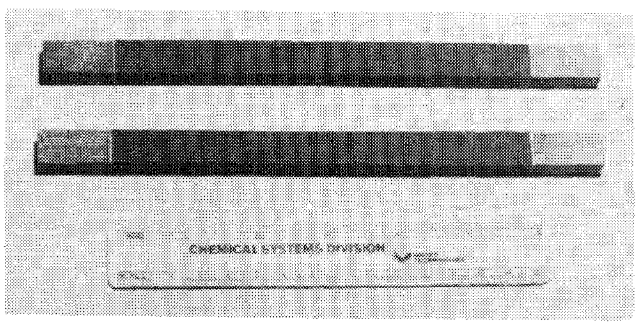

Fig. 1 Tensile specimen of solid propellant.

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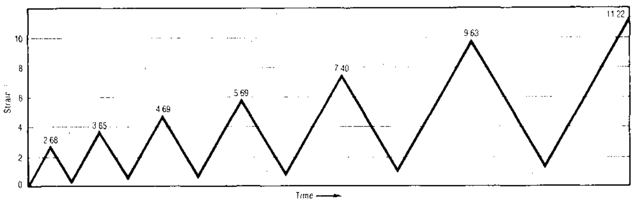

Fig. 2 Strain history.

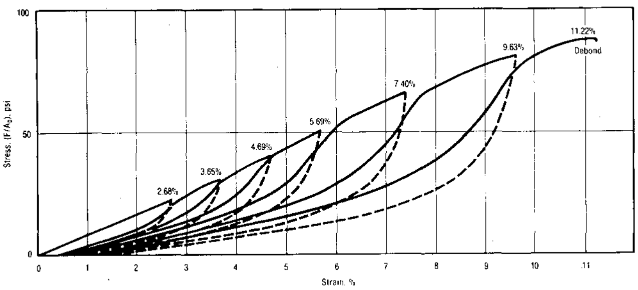

Fig. 3 Stress-strain response.

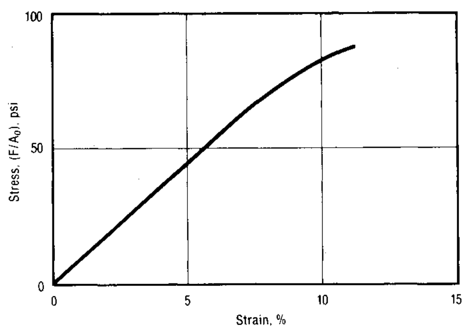

Fig. 4 Virgin stress-strain response.

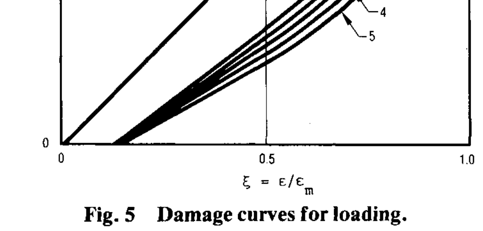

Fig. 5 Damage curves for loading.

End-bonded test specimens of dimensions $6 \times 0.5 \times 0.5$ in.[3] were placed on an Instron test machine and loaded using the strain history shown in Fig. 1. The test was performed at 70°F and a strain rate of 0.03 min^{-1}. The values of maximum strain for loading were selected to increase the strain level by 1% strain or more on each cycle. On unloading, the crosshead direction was reversed each time the stress reached zero.

The stress response for the six-cycle history of Fig. 1 is presented in Fig. 2. Note that in accord with the earlier hypothesis, the stress returns to the virgin curve once the previous maximum strain has been exceeded. The unloading-reloading curves show the same form for all strain levels.

The *virgin curve* is shown in Fig. 3. This curve was established by taking the envelope of the curves in Fig. 2 and also by performing a separate test in which the strain history was increased monotonically (at the same strain rate); the two curves coincide almost exactly. Interestingly the virgin curve demonstrates linear response up to 7% strain.

The unloading curves in Fig. 2 were normalized to generate the damage curves of Eq. (7). These appear to form a family of curves that depend on the strain level but intersect on the ξ axis at a value of ξ approximately equal to 0.14.

While the damage curves in Fig. 4 do not collapse to a single curve, for some applications, it might be reasonable to approximate the resulting family of curves by a single master damage curve as represented by Eq. (9).

It should be noted that in Fig. 4 only the unloading curves are graphed. The discrepancy in Fig. 2 between loading and unloading below the maximum strain level is due to rate effects and is less pronounced at lower strain rates. Of course, since this theory is rate independent, it cannot account for effects such as these without modification.

Finally, it should be remarked that graphs of the virgin curves and damage curves, such as those shown in Figs. 3 and 4, seem to yield an organized method of cataloging the stress-strain behavior of highly filled solid propellants. This procedure may also be useful in studies of other materials that undergo damage.

Acknowledgment

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References

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Void Characteristics of a Liquid-Filled Cylinder Undergoing Spinning and Coning Motion

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RECENT studies have revealed a new type of flight instability experienced by spin-stabilized projectiles having liquid fills of relatively high viscosity.[1] The instability is characterized by a growth in projectile yaw angle and a concurrent loss in projectile spin with a consequent

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